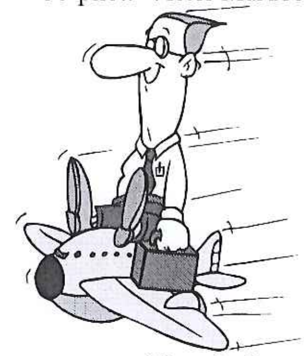
AIPLANE Pilot : Captn. Roger Over
Co-pilot: Victor Murdock



What's your vector Victor? -Over
Captain Over- heading at vector 4-9EER-3 - Over
Roger, Roger, Victor – Over

Honors Physics

Ch 3. Vectors

<u>Lesson 1: Vectors in 1-D</u>

Vectors - have magnitude and direction

Scalars - have magnitude only

Notation

$\rightharpoonup$ Vector symbol $\quad \vec{A}$ = Vector

A = Scalar

$\wedge$ Unit vector $\quad \hat{x}$ = 1 unit in the x-direction

Vectors - Graphical Representation

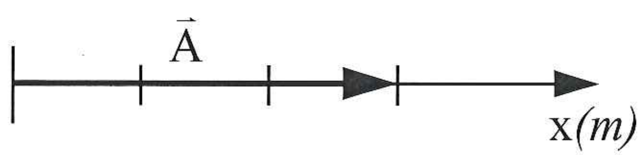
Vectors can be represented using "<u>arrows</u>"

Length of the arrow = magnitude

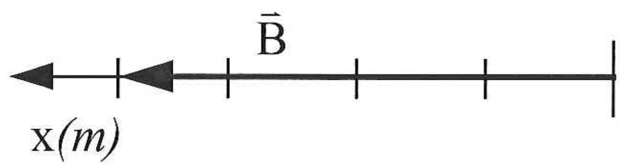
Direction the arrow points = direction (given by unit vector or angle θ)

Example: "Positions represent vectors because they are defined relative to an origin.

$\vec{A} = +3 \text{ m } \hat{x}$



$\vec{B} = -4 \text{ m } \hat{x}$



Positions have distance (magnitude) from the origin and they have direction from the origin.

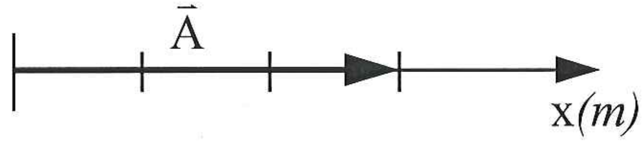
Vector Addition in 1-D

$\vec{A} + \vec{B} = \vec{C}$

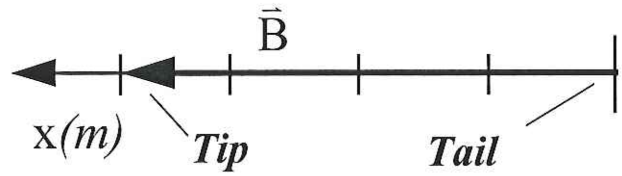
The sum of two vectors is a vector, $\vec{C}$ is called the resultant vector of the vector addition.

Graphical Method for adding vectors

$\vec{A} = +3 \text{ m } \hat{x}$



$\vec{B} = -4 \text{ m } \hat{x}$



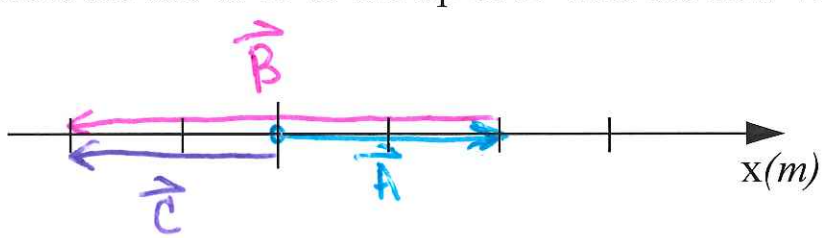
Say you want to find vector $\vec{C} = \vec{A} + \vec{B}$

Tip to Tail Method – Graphical Approach – Use Ruler

1. Take the tail of vector $\vec{B}$ and connect it to the tip of vector $\vec{A}$

(Note: do not change the orientation of $\vec{B}$ just "pick it up" and "pin it down")

2. Connect the tail of $\vec{A}$ to the tip of $\vec{B}$ with the new vector $\vec{C}$



Vector $\vec{C}$ is called the ***resultant vector***

Algebraic Method – Simple Addition

Say you want to find vector $\vec{C} = \vec{A} + \vec{B}$

And
$$\vec{A} = +150\ m\ \hat{x}$$
$$\vec{B} = -25\ m\ \hat{x}$$

You can always + or – vectors that are along the same axis the same as you would + or – numbers:

$\therefore$
$$\begin{array}{r} \vec{A} = +\ 150\, m\, \hat{x} \\ +\ \vec{B} = -\ 25\ m\, \hat{x} \\ \hline \vec{C} = 125 m \hat{x} \end{array}$$

$$\begin{array}{r} \vec{A} = +\ 573.5\, m\, \hat{y} \\ +\ \vec{B} = -\ 43.25\ m\, \hat{y} \\ \hline \vec{C} = 530.25 m \hat{y} \end{array}$$

You can have fractions/decimals in vectors.

Vector Subraction in 1-D

$$\vec{A} - \vec{B} = \vec{C}$$

Vector Subtraction is the same vector addition but now you must add the negative of the vector

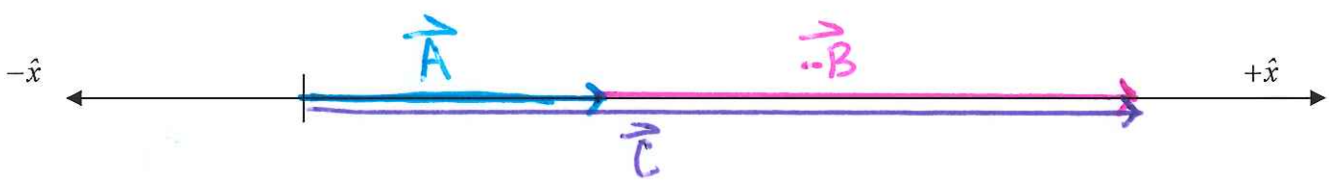
$$\vec{A} + \left(-\vec{B}\right) = \vec{C}$$

Take the opposite of vector $\vec{B}$, then add!

Example: Use 1cm for 1 unit below.

$\vec{A} = 4\,\hat{x}$ $\quad\quad$ $\vec{B} = -7\,\hat{x}$ $\quad\quad$ $\vec{C} = \vec{A} - \vec{B}$

$-\vec{B} = 7\hat{x}$



Algebraic vector subtraction in 1 dimension is just like regular subtraction of numbers.

$$\vec{A} = +\,4\,\hat{x}$$

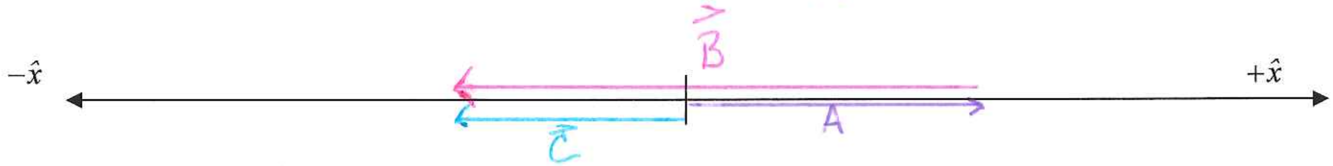
$$-\;\vec{B} = -(-7)\,\hat{x}$$

$$\vec{C} = 11\hat{x}$$

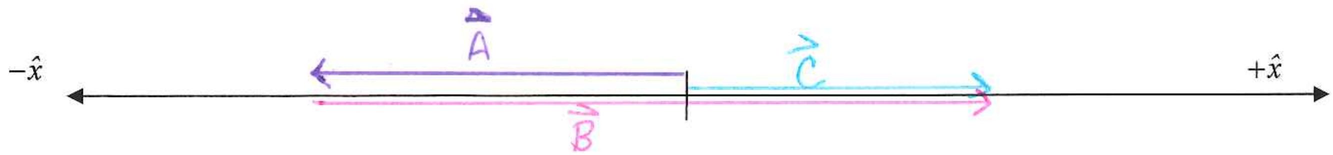
Lesson 1 Problems: Vectors in 1-D

1. Use both the tip-to-tail and the algebraic method to determine the Resultant Vector. Use the scale that 1 cm = 1 unit for the given vectors

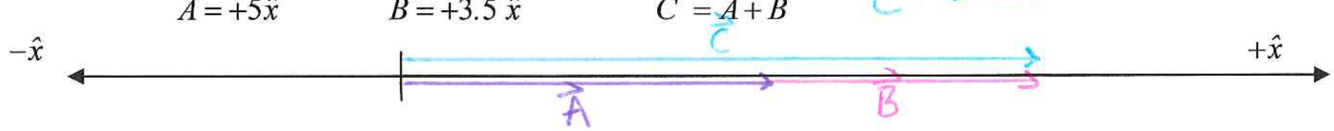
$\vec{A} = 4\,\hat{x}$ $\quad \vec{B} = -7\,\hat{x}$ $\quad \vec{C} = \vec{A}+\vec{B} = -3\hat{x}$



$\vec{A} = -5\hat{x}$ $\quad \vec{B} = +9\,\hat{x}$ $\quad \vec{C} = \vec{A}+\vec{B}$ $\quad \vec{C} = 4\hat{x}$



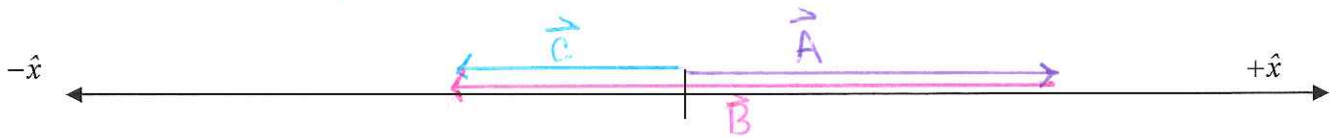
$\vec{A} = +5\hat{x}$ $\quad \vec{B} = +3.5\,\hat{x}$ $\quad \vec{C} = \vec{A}+\vec{B}$ $\quad \vec{C} = 8.5\hat{x}$



2. Use both the tip-to-tail and the algebraic method to determine the Resultant Vector. Use the scale that 1 cm = 1 unit for the given vectors

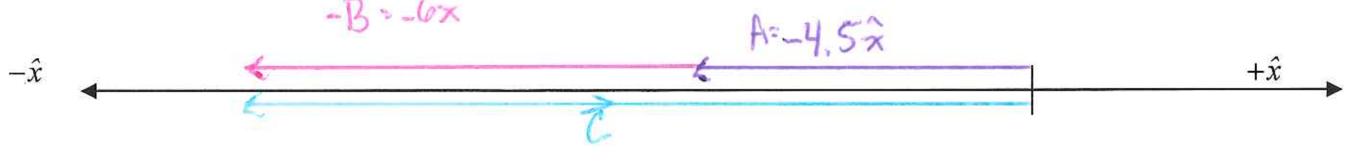
$\vec{A} = +5\hat{x}$ $\quad$ $\vec{B} = +8\,\hat{x}$ $\quad$ $\vec{C} = \vec{A} - \vec{B}$ $\quad$ $\vec{C} = -3\hat{x}$

$-\vec{B} = -8\hat{x}$



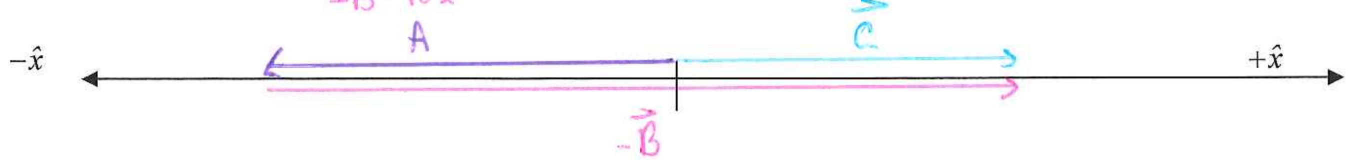
$\vec{A} = -4.5\hat{x}$ $\quad$ $\vec{B} = +6\,\hat{x}$ $\quad$ $\vec{C} = \vec{A} - \vec{B}$ $\quad$ $\vec{C} = -10.5\hat{x}$

$-\vec{B} = -6\hat{x}$



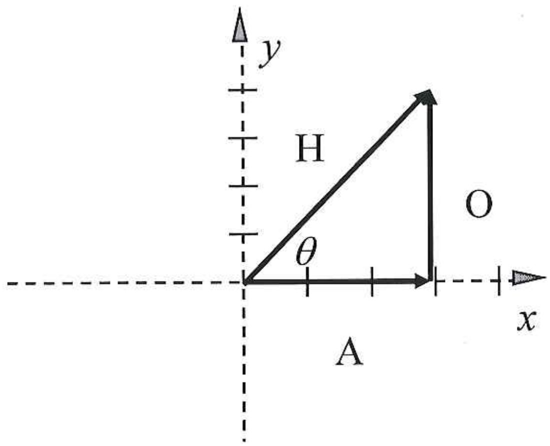
$\vec{A} = -5.5\hat{x}$ $\quad$ $\vec{B} = -10\,\hat{x}$ $\quad$ $\vec{C} = \vec{A} - \vec{B}$ $\quad$ $\vec{C} = 4.5\hat{x}$

$-\vec{B} = 10\hat{x}$



Lesson 2: Vector Components in 2-D

Trigonometry Review – Right Triangle Mathematics is critical for working with vectors in 2 Dimensions.



$$H^2 = O^2 + A^2$$

$$\sin\theta = \frac{\mathrm{O}}{\mathrm{H}}$$

$$\theta = \tan^{-1}\left(\frac{O}{A}\right)$$

$$\cos\theta = \frac{\mathrm{A}}{\mathrm{H}}$$

"SOH CAH TOA"

$$\tan\theta = \frac{\mathrm{O}}{\mathrm{A}}$$

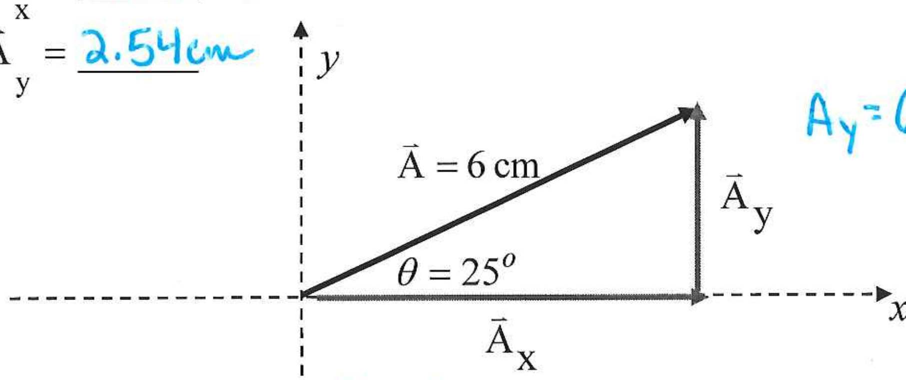
Vector Components in 2-D

☆ **Any 2-D vector can be broken down into two component vectors such that**

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$\vec{A}_x$ = 5.44cm

$\vec{A}_y$ = 2.54cm



$A_y = 6\sin 25 = 2.54\text{cm}$

$A_x = 6\cos 25$

☆ Vector **A** written in **Component Form** is sometimes called **Cartesian Form.** Vector **A** written as a length and an angle is called **Magnitude and Direction Form** or **Polar Form**

Component form:

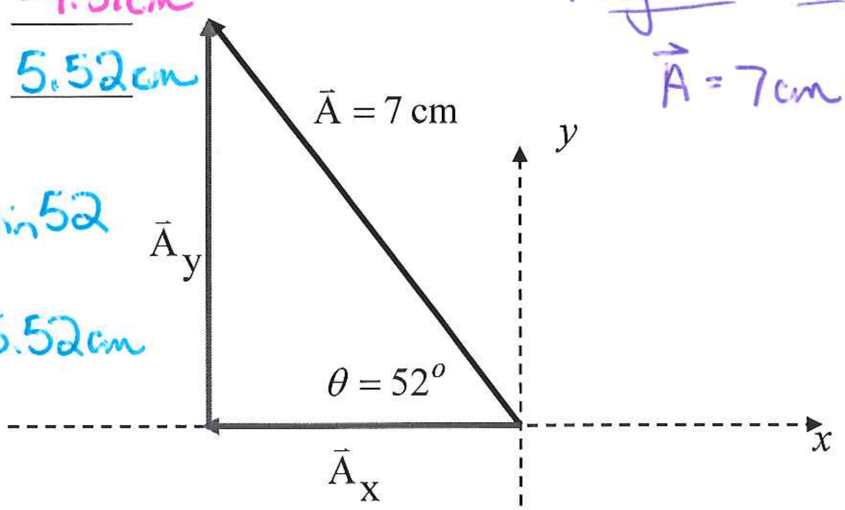
$\vec{A} = -4.31\text{cm}\,\hat{x} + 5.52\text{cm}\,\hat{y}$

Magnitude and direction form:

$\vec{A}$ = 7cm @ 52° up from $-\hat{x}$

$\vec{A}_x$ = -4.31cm

$\vec{A}_y$ = 5.52cm



$A_y = 7\sin 52$

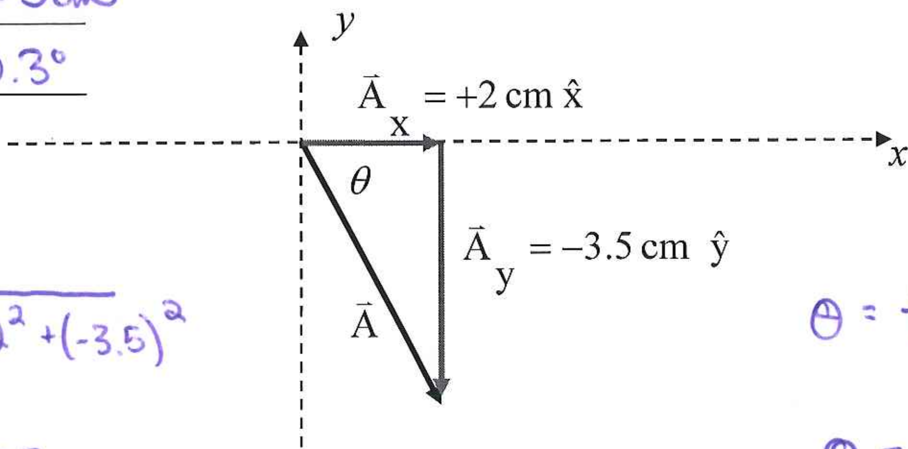
$A_y = 5.52\text{cm}$

$A_x = 7\cos 52 = 4.31\text{ cm}$

Given the component vectors you can find the vector that they make and the angle θ.

$\vec{A} =$ 4.03cm

$\theta =$ 60.3°



$\vec{A} = \sqrt{2^2 + (-3.5)^2}$

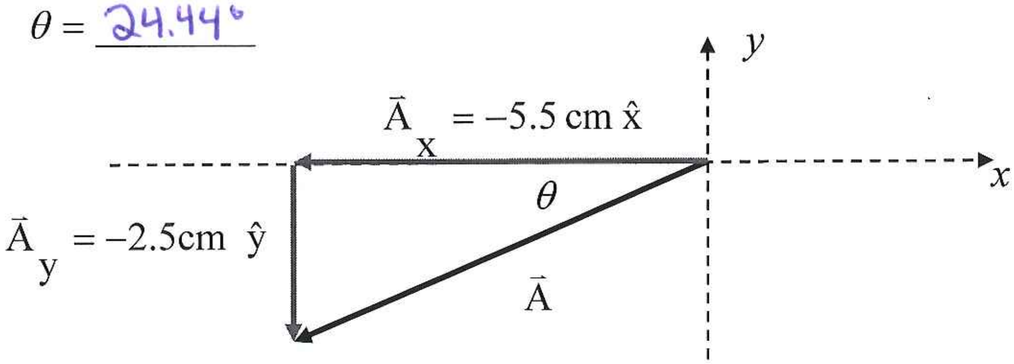
$\vec{A} = 4.03\text{cm}$

$\theta = \tan^{-1}\left(\frac{-3.5}{2}\right)$

$\theta = 60.3°$

$\vec{A} =$ 6.0415cm

$\theta =$ 24.44°



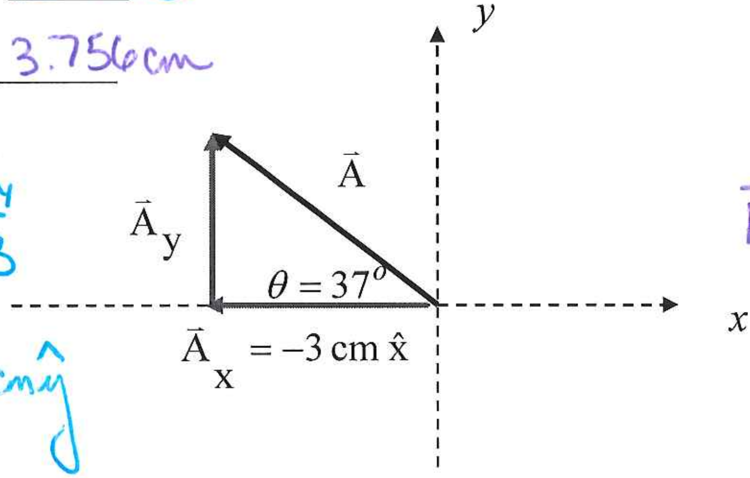
$\vec{A} = \sqrt{-5.5^2 + -2.5^2} = 6.0415\text{cm}$

$\theta = \tan^{-1}\left(\frac{2.5}{5.5}\right) = 24.44°$

Given one component vector and the angle θ you can find the other component vector and the vector itself.

$\vec{A}_y$ = 2.26cm

$\vec{A}$ = 3.756cm



$\tan 37 = \frac{\vec{A}_y}{-3}$

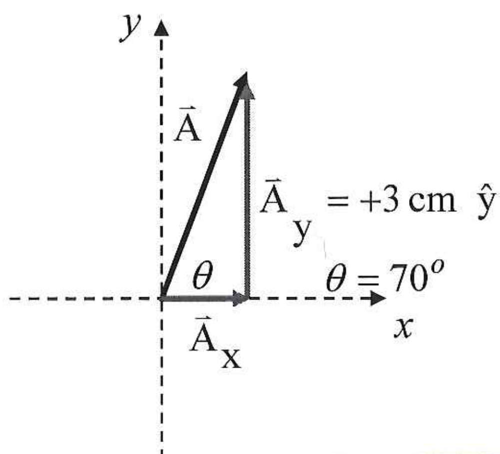
$\vec{A}_y = 2.26\text{cm}\,\hat{y}$

$\vec{A} = \sqrt{3^2 + 2.26^2}$

$\vec{A} = 3.756\text{ cm}$

$\vec{A}_x$ = 1.0919cm$\hat{x}$

$\vec{A}$ = 3.1925cm



$\tan 70 = \frac{3}{A_x}$

$A_x = 1.0919\text{cm}\,\hat{x}$

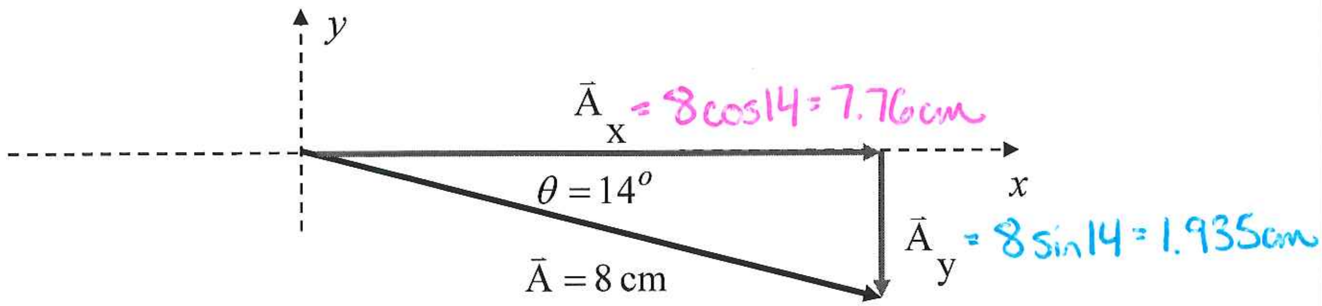
$\vec{A} = \sqrt{3^2 + 1.0919^2} = 3.1925\text{cm}$

Lesson 2 Problems: Vectors in 2-D

1. Find the component vectors given the information below

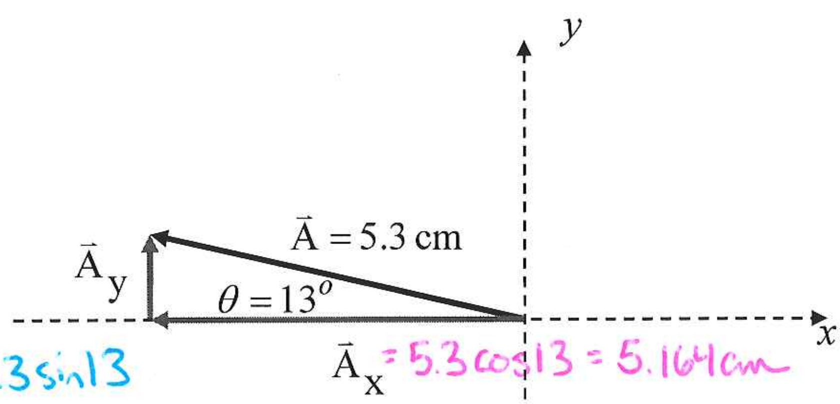
$\vec{A}_x$ = 7.76cm $\hat{x}$

$\vec{A}_y$ = -1.935cm $\hat{y}$



$\vec{A}_x$ = -5.164 cm

$\vec{A}_y$ = 1.192cm



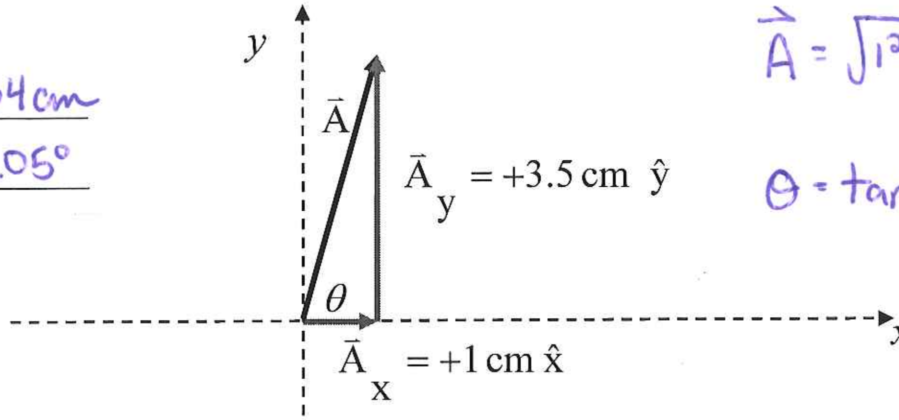
A_y = 5.3sin13

= 1.192cm

2. Find the Vector and the Angle shown below.

$\vec{A} =$ 3.64 cm

$\theta =$ 74.05°

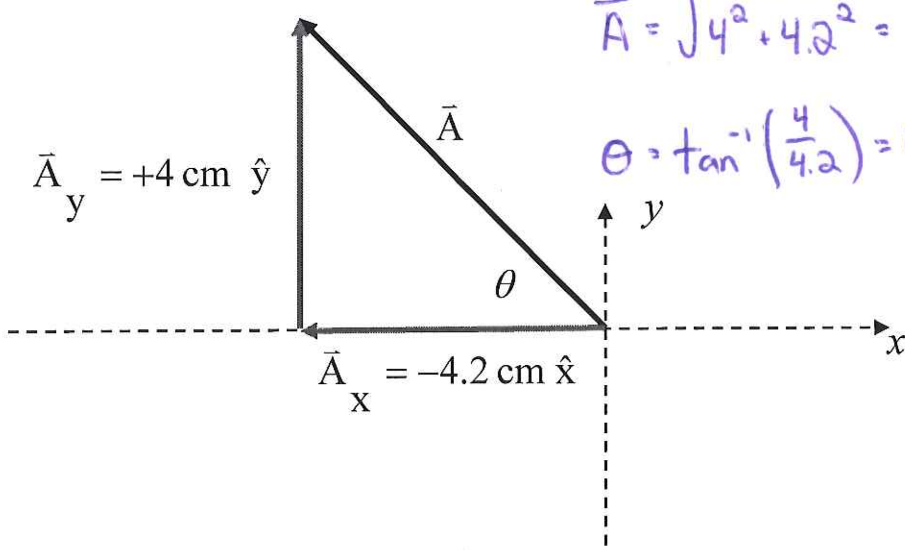


$\vec{A} = \sqrt{1^2 + 3.5^2} = 3.64\text{ cm}$

$\theta = \tan^{-1}\left(\frac{3.5}{1}\right) = 74.05°$

$\vec{A} =$ 5.8 cm

$\theta =$ 43.6°



$\vec{A} = \sqrt{4^2 + 4.2^2} = 5.8\text{ cm}$

$\theta = \tan^{-1}\left(\frac{4}{4.2}\right) = 43.6°$

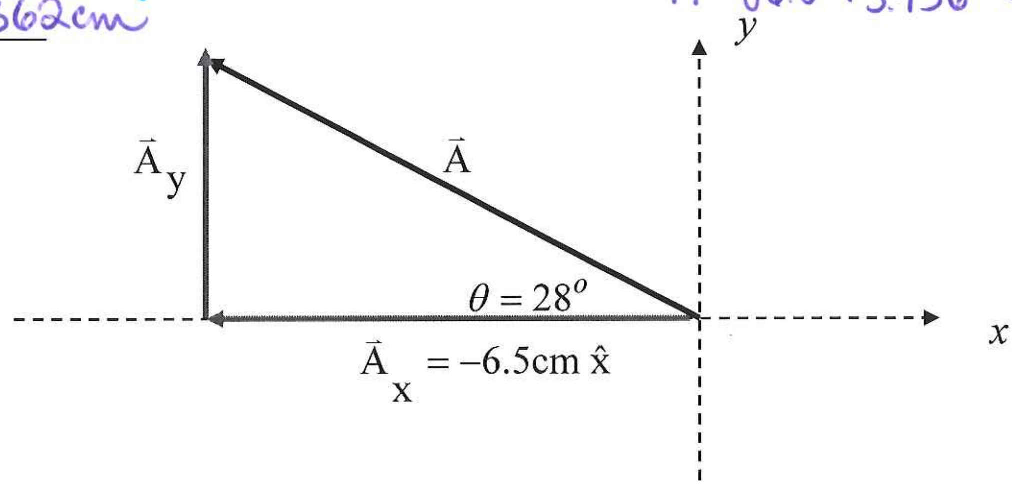
3. Find the component vector and the resultant vector.

$\tan 28 = \frac{A_y}{6.5} \Rightarrow A_y = 3.456\text{cm}$

$\vec{A}_y = $ 3.456cm $\hat{y}$

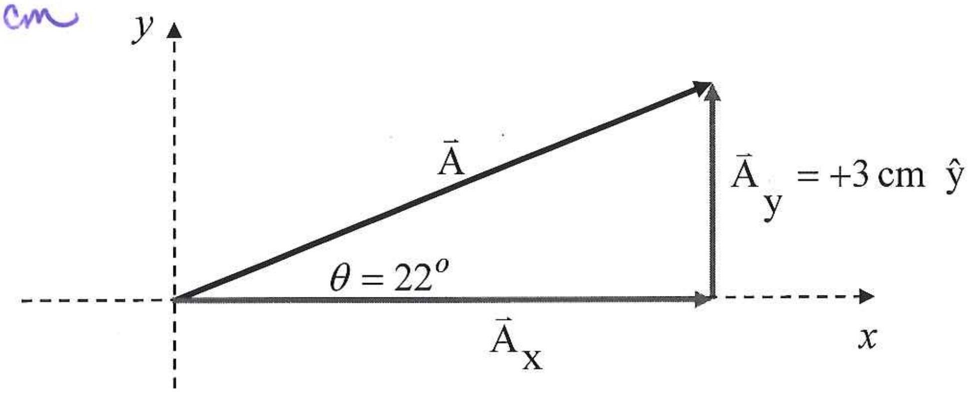
$\vec{A} = $ 7.362cm

$A = \sqrt{6.5^2 + 3.456^2} = 7.362\text{cm}$



$\vec{A}_x = $ 7.425cm

$\vec{A} = $ 8.008cm



$\tan 22 = \frac{3}{A_x} \rightarrow A_x = 7.425\text{cm}$

$\vec{A} = \sqrt{3^2 + 7.425^2} = 8.008\text{cm}$

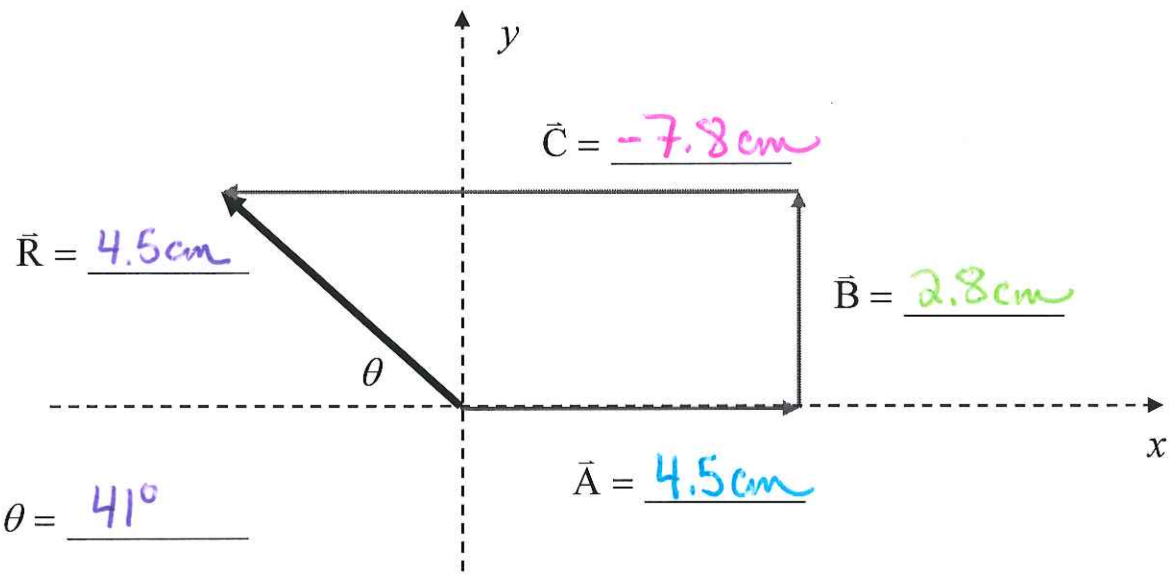
Lesson 3: Vector Addition in 2-D

Tip to Tail Method – Ruler and Protractor

Measure vectors **A, B** and **C** below in **cm** and write them with unit vectors to show direction.

Measure the vector **R** with a ruler and write down its length.

Measure the angle **θ** with a protractor and write it down



$$\vec{R}=\vec{A}+\vec{B}+\vec{C}$$

Now confirm that when you add and subtract the component vectors you get the correct vector **R** in **Component Form**.

$\vec{A}=$ 4.5cm $\hat{x}+$ 0 $\hat{y}$

$\vec{B}=$ 0 $\hat{x}+$ 2.8cm $\hat{y}$

$\vec{C}=$ -7.8cm $\hat{x}+$ 0 $\hat{y}$

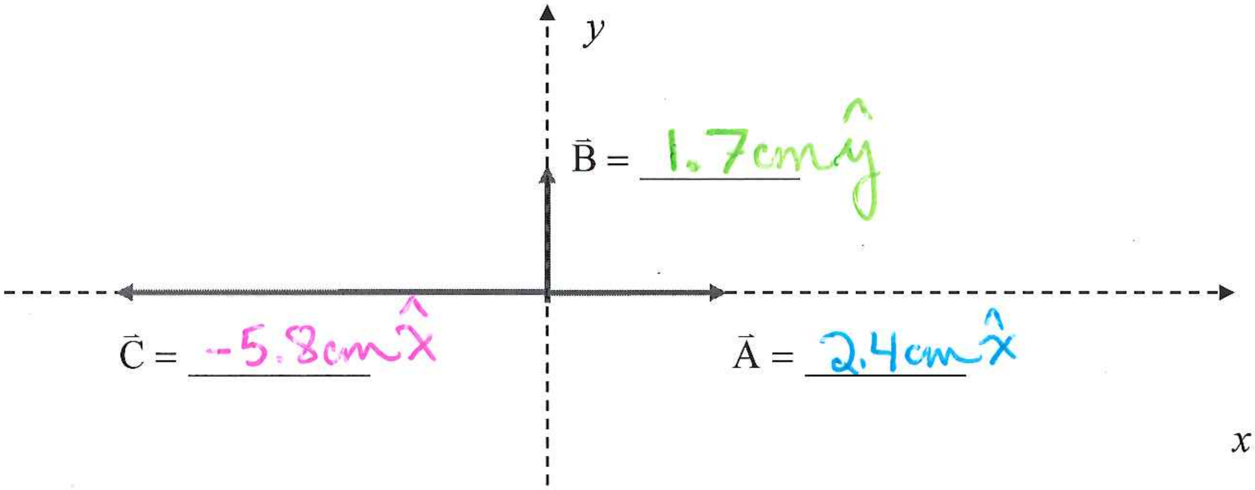
$\vec{R}=$ -3.3cm $\hat{x}+$ 2.8cm $\hat{y}$

check:

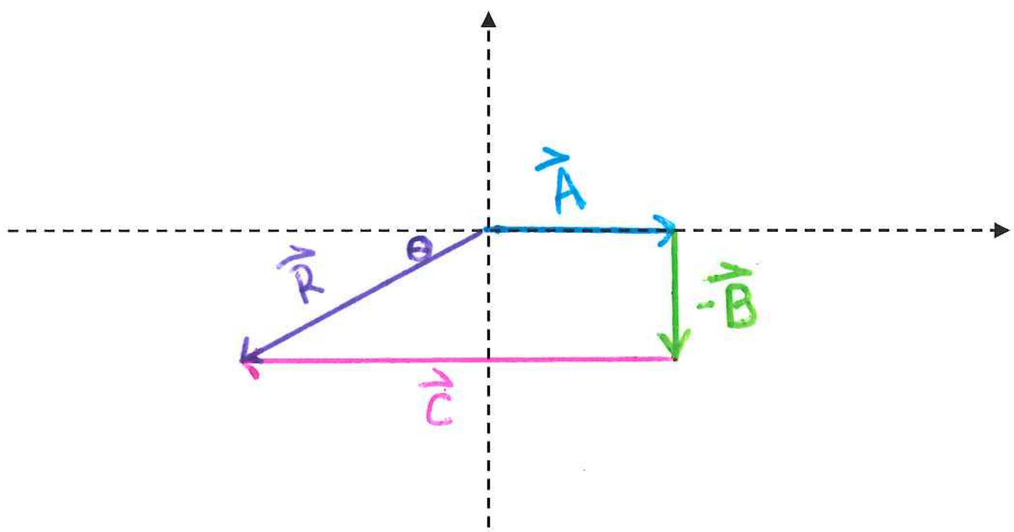
$\vec{R} = \sqrt{3.3^2 + 2.8^2} = 4.33\text{cm}$

$\theta = \tan^{-1}\left(\frac{2.8}{3.3}\right) = 40.3°$

Measure vectors **A, B** and **C** below in **cm** and write them with unit vectors to show direction.



Using a ruler and protractor show the tip-to-tail vector addition **R=A−B+C** below. Draw in the vector **R** and measure the angle θ it makes with the negative x axis.



Now confirm that when you add and subtract the component vectors you get the correct vector **R** in **Component Form**.

$\vec{A}$ = 2.4cm $\hat{x}$ + 0 cm $\hat{y}$

$\vec{B}$ = 0cm $\hat{x}$ + -1.7cm $\hat{y}$

$\vec{C}$ = -5.8cm $\hat{x}$ + 0cm $\hat{y}$

$\vec{R}$ = -3.4cm $\hat{x}$ + -1.7cm $\hat{y}$

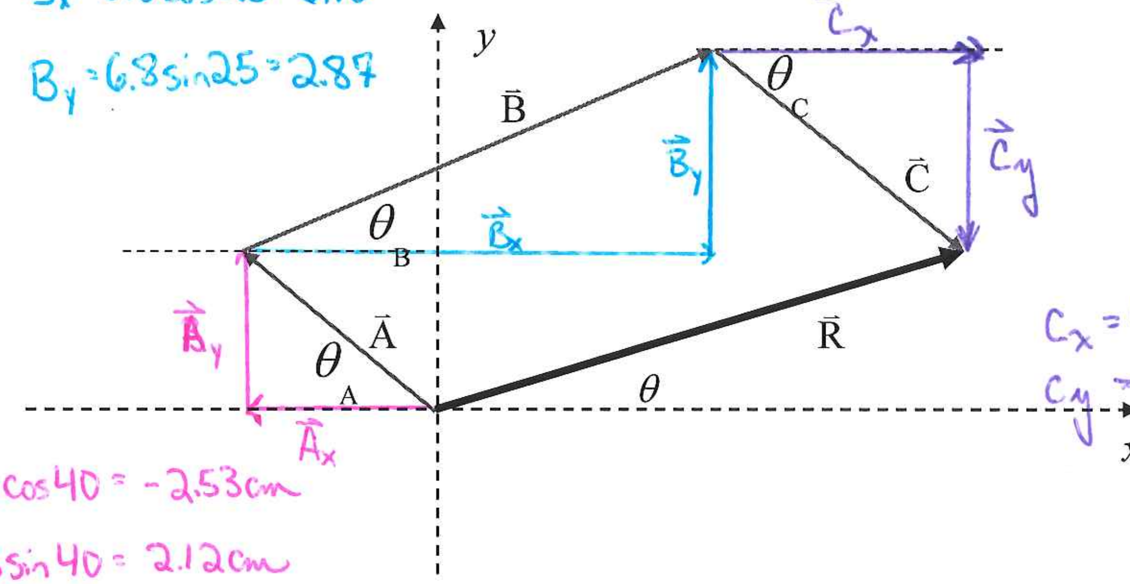
$\vec{R} = \sqrt{1.7^2 + 3.4^2} = 3.8$ cm

$\theta = \tan^{-1}\left(\frac{1.7}{3.4}\right) = 26.565°$

$\vec{R} = 3.8$cm @ $26.6°$

$B_x = 6.8\cos 25 = 6.16$

$B_y = 6.8\sin 25 = 2.87$



$C_x = 4.3\cos 38 = 3.39$

$C_y = 4.3\sin 38 = 2.65$

$A_x = 3.3\cos 40 = -2.53\text{cm}$

$A_y = 3.3\sin 40 = 2.12\text{cm}$

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

Using a Ruler and Protractor write each of the vectors in Polar Form

$\vec{A} =$ 3.3 cm, $\theta_A =$ 40°

$\vec{B} =$ 6.8 cm, $\theta_B =$ 25°

$\vec{C} =$ 4.3 cm, $\theta_C =$ 38°

$\vec{R} =$ 7.4 cm, $\theta =$ 17°

Using Right Triangle Math calculate the following components and verify the addition of the 3 vectors in component form corresponds to the polar form above.

$\vec{A} =$ -2.53 $\hat{x}$ + 2.12 $\hat{y}$

$\vec{B} =$ 6.16 $\hat{x}$ + 2.87 $\hat{y}$

$\vec{C} =$ 3.39 $\hat{x}$ + -2.65 $\hat{y}$

$\vec{R} =$ 7.02 $\hat{x}$ + 2.34 $\hat{y}$

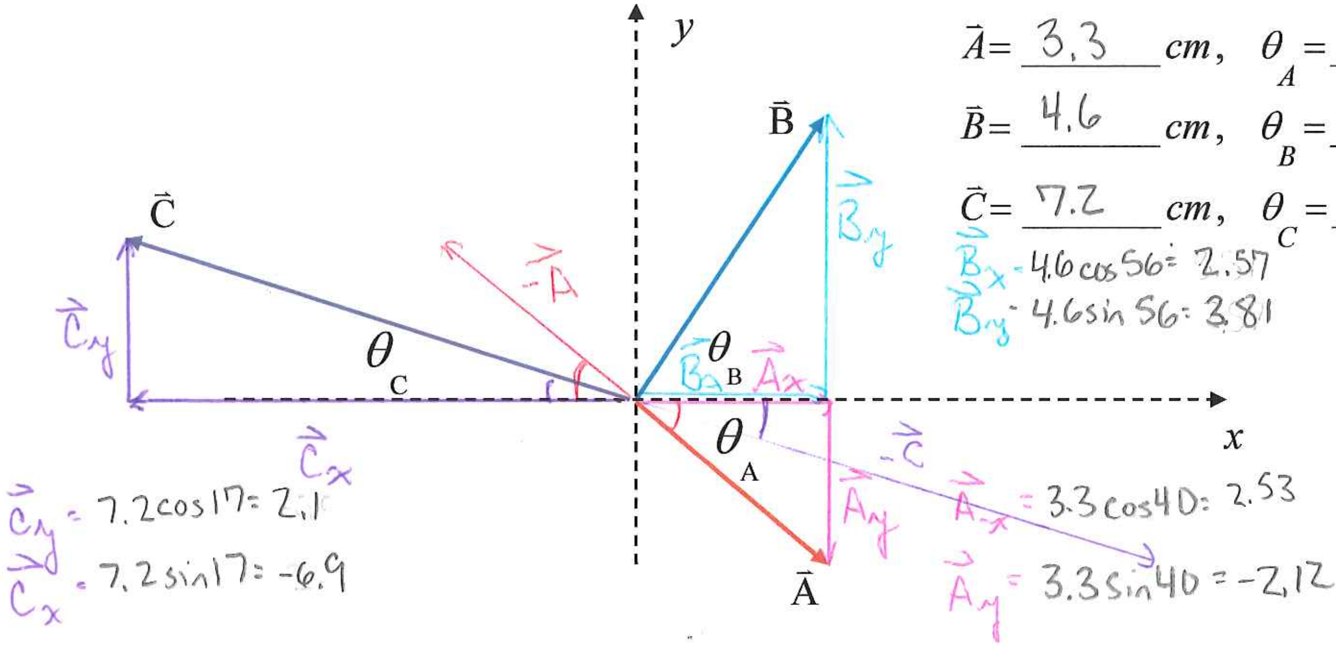
check:

$\vec{R} = \sqrt{7.02^2 + 2.34^2} =$

$\vec{R} = 7.4\text{cm}$ ✓

$\theta = \tan^{-1}\left(\frac{2.34}{7.02}\right) = 18.4°$ ✓

Using a ruler and protractor write down the vectors in Polar Form



$\vec{A}=$ 3.3 cm, $\theta_A =$ 40°

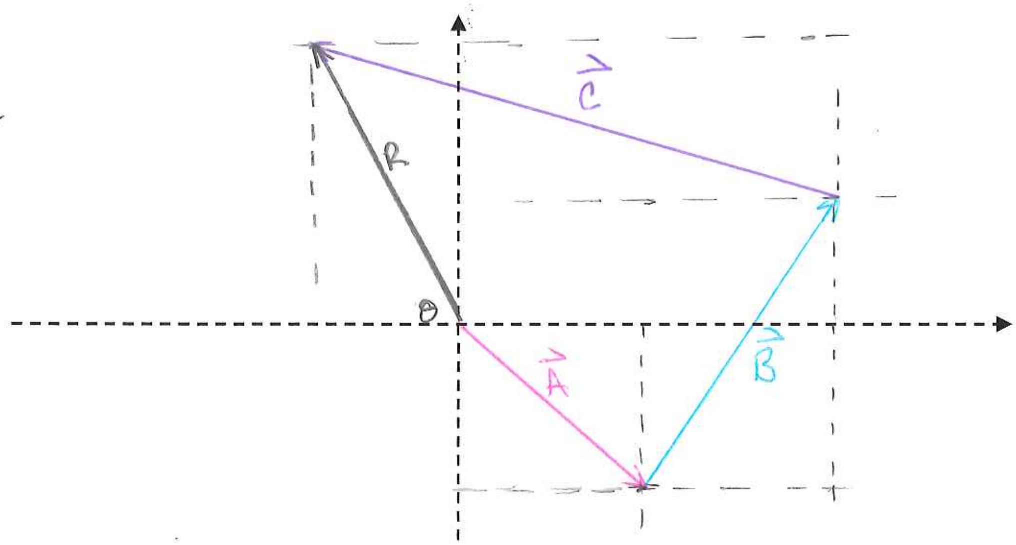
$\vec{B}=$ 4.6 cm, $\theta_B =$ 56°

$\vec{C}=$ 7.2 cm, $\theta_C =$ 17°

Using a ruler and protractor show the tip-to-tail vector addition **R=A+B+C** below. Draw in the vector **R** and measure the angle θ it makes with the negative x axis.

$\vec{R} = 4.2cm$

$\theta = 63°$



Calculate the component form of each of the vectors and confirm that the vector R is the same as

$\vec{A} =$ 2.53 $\hat{x}$ + ~~-2.12~~ 2.12 $\hat{y}$

$\vec{B} =$ 2.57 $\hat{x}$ + 3.81 $\hat{y}$

$\vec{C} =$ -6.9 $\hat{x}$ + 2.1 $\hat{y}$

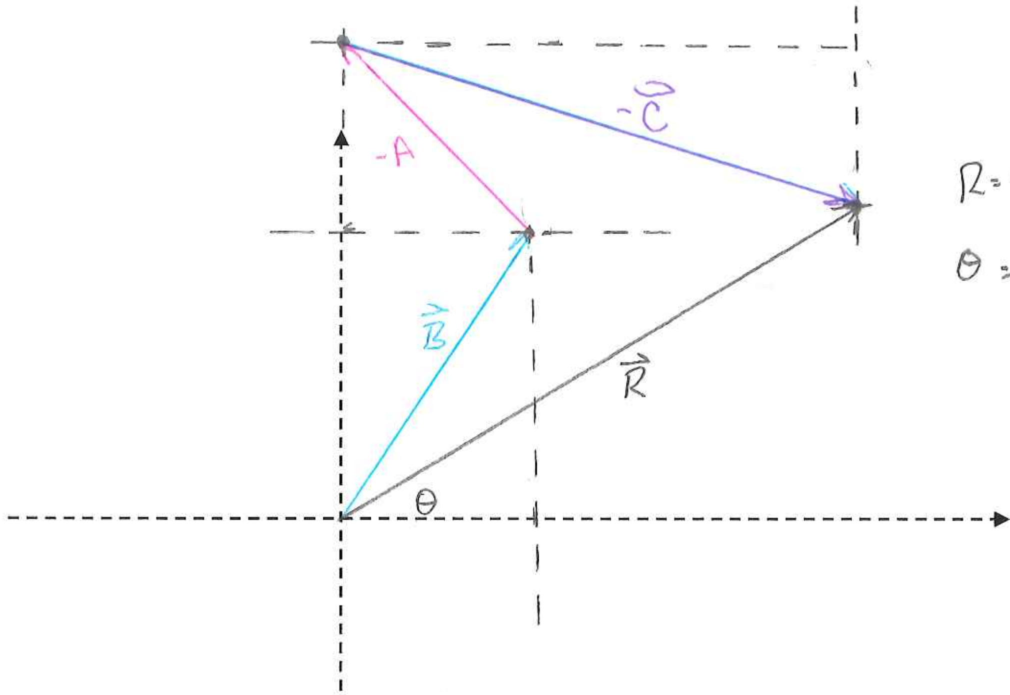
$\vec{R} =$ -1.8 $\hat{x}$ + 3.79 $\hat{y}$

$R = \sqrt{1.8^2 + 3.79^2} = 4.2cm$

$\theta = \tan^{-1}\left(\frac{3.79}{1.8}\right)$

$\theta = 64.6°$

Using a ruler and protractor show the tip-to-tail vector addition **R=B−A−C** below. Draw in the vector **R** and measure the angle θ it makes with the positive x axis.



R = 8cm

θ = 29°

Calculate the component form of each of the vectors and confirm that the vector R is the same as

$\vec{A}=$ -2.53 $\hat{x}$ + 2.12 $\hat{y}$

$\vec{B}=$ 2.57 $\hat{x}$ + 3.81 $\hat{y}$

$-\vec{C}=$ 6.91 $\hat{x}$ + -2.1 $\hat{y}$

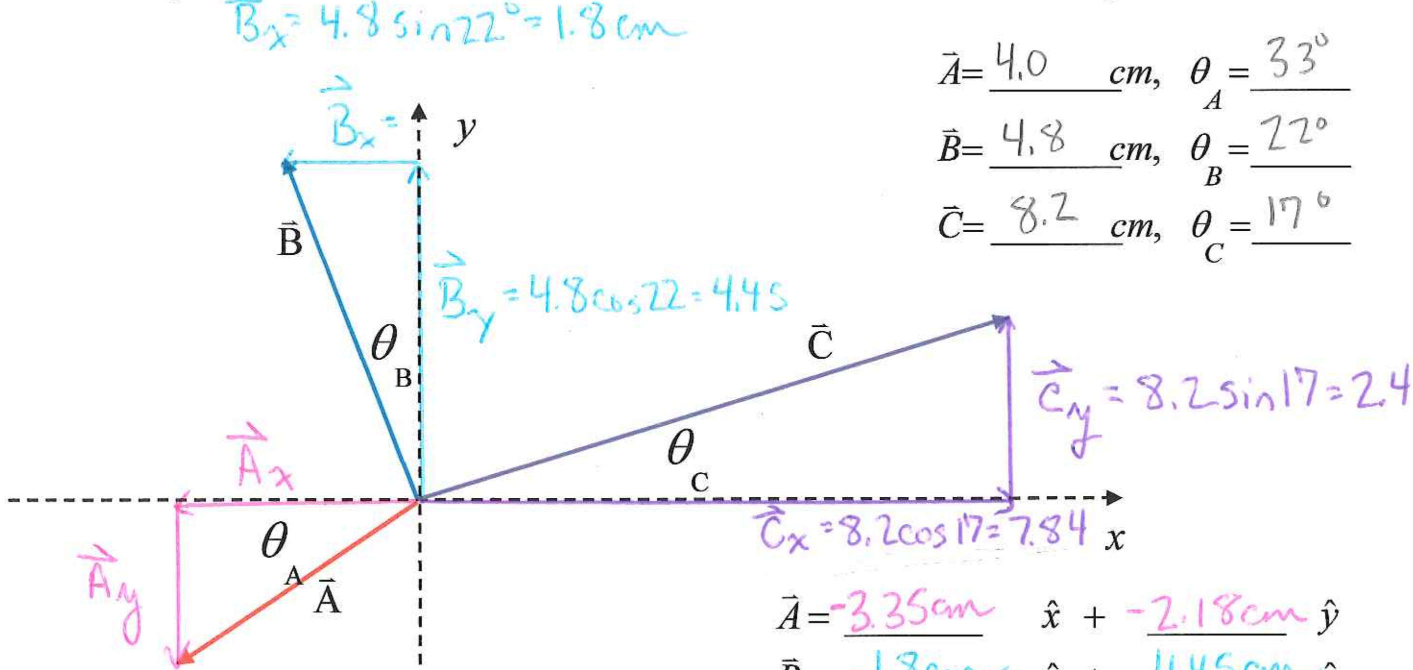
$\vec{R}=$ 6.94 $\hat{x}$ + 3.83 $\hat{y}$

$\vec{R} = \sqrt{6.94^2 + 3.83^2} = 7.72 \text{ cm}$

$\theta = \tan^{-1}\left(\frac{3.83}{6.94}\right) = 28.89^{\circ}$

Lesson 3 Problems: Vector Addition in 2-D

1. Using a Ruler and Protractor write each of the vectors in Polar Form. Then calculate the component form of each of the vectors and calculate that the vector **R** in Component Form.



$\vec{A}=$ 4.0 cm, $\theta_A =$ 33°

$\vec{B}=$ 4.8 cm, $\theta_B =$ 22°

$\vec{C}=$ 8.2 cm, $\theta_C =$ 17°

$\vec{A} =$ -3.35cm $\hat{x}$ + -2.18cm $\hat{y}$

$\vec{B} =$ -1.8cm $\hat{x}$ + 4.45cm $\hat{y}$

$\vec{C} =$ 7.84cm $\hat{x}$ + 2.4 cm $\hat{y}$

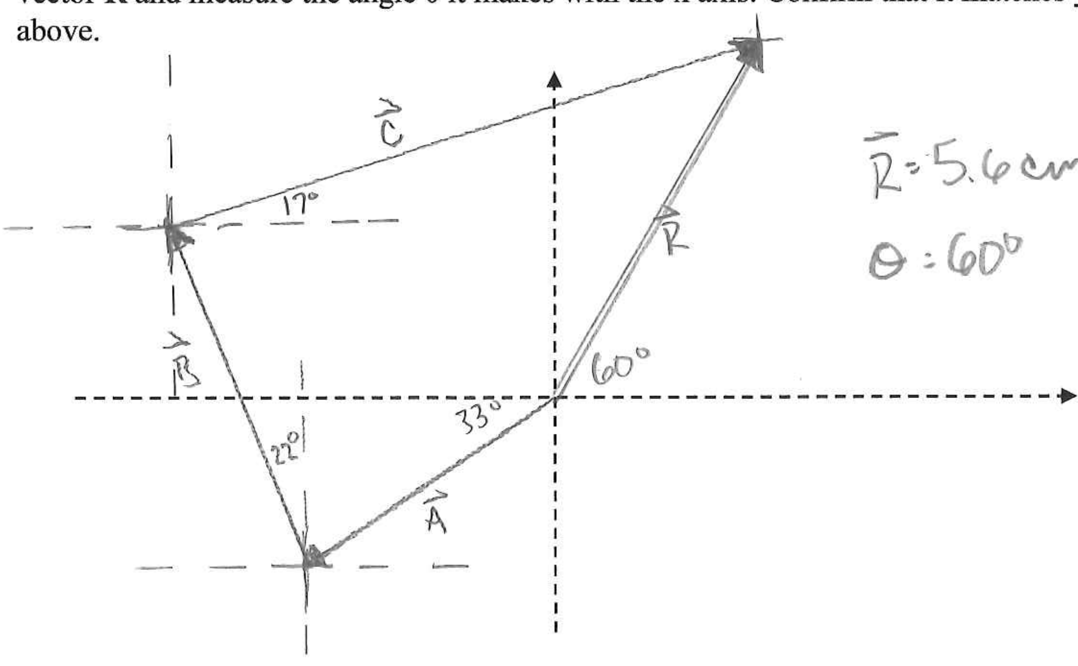
$\vec{R} =$ 2.69 $\hat{x}$ + 4.67 $\hat{y}$

$$\vec{R}=\vec{A}+\vec{B}+\vec{C}$$

$\vec{A}_x = 4 \cos 33 = 3.35$ cm $\hat{x}$

$\vec{A}_y = 4 \sin 33 = 2.18$ cm $\hat{y}$

Using a ruler and protractor show the tip-to-tail vector addition of **A** + **B** + **C** below. Draw in the vector **R** and measure the angle θ it makes with the x axis. Confirm that it matches your answer above.



$\vec{R} = 5.6$ cm

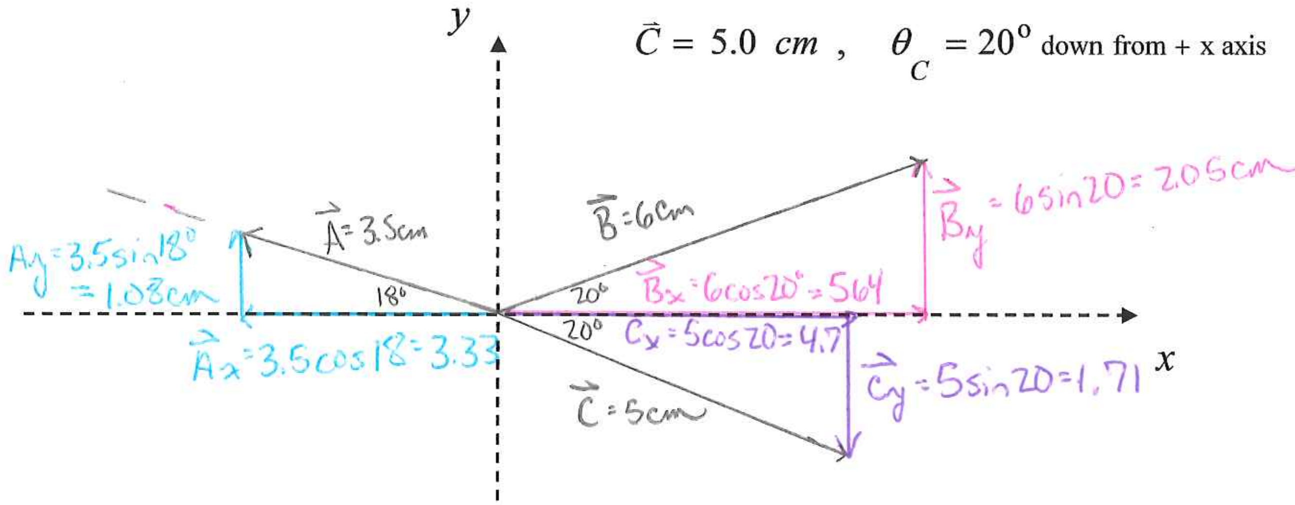
$\theta = 60°$

2. Draw in the following vectors below.

$\vec{A} = 3.5\ cm\ ,\quad \theta_A = 18^{o}$ up from – x axis

$\vec{B} = 6.0\ cm\ ,\quad \theta_B = 20^{o}$ up from + x axis

$\vec{C} = 5.0\ cm\ ,\quad \theta_C = 20^{o}$ down from + x axis



Calculate the component form of each of the vectors and calculate that the vector **R=A+B+C** in Component Form.

$\vec{A}=$ -3.33 $\hat{x}$ + 1.08cm $\hat{y}$

$\vec{B}=$ 5.64cm $\hat{x}$ + 2.05cm $\hat{y}$

$\vec{C}=$ 4.7cm $\hat{x}$ + -1.71 $\hat{y}$

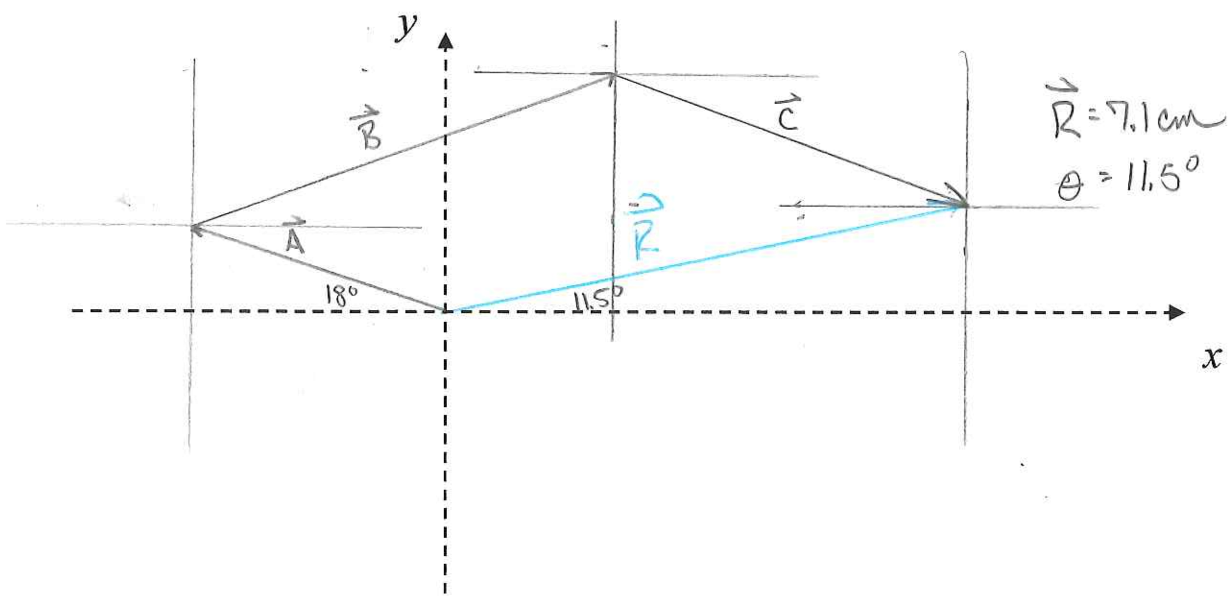
$\vec{R}=$ 7.01 $\hat{x}$ + 1.42 $\hat{y}$

$\vec{R} = \sqrt{7.01^2 + 1.42^2}$

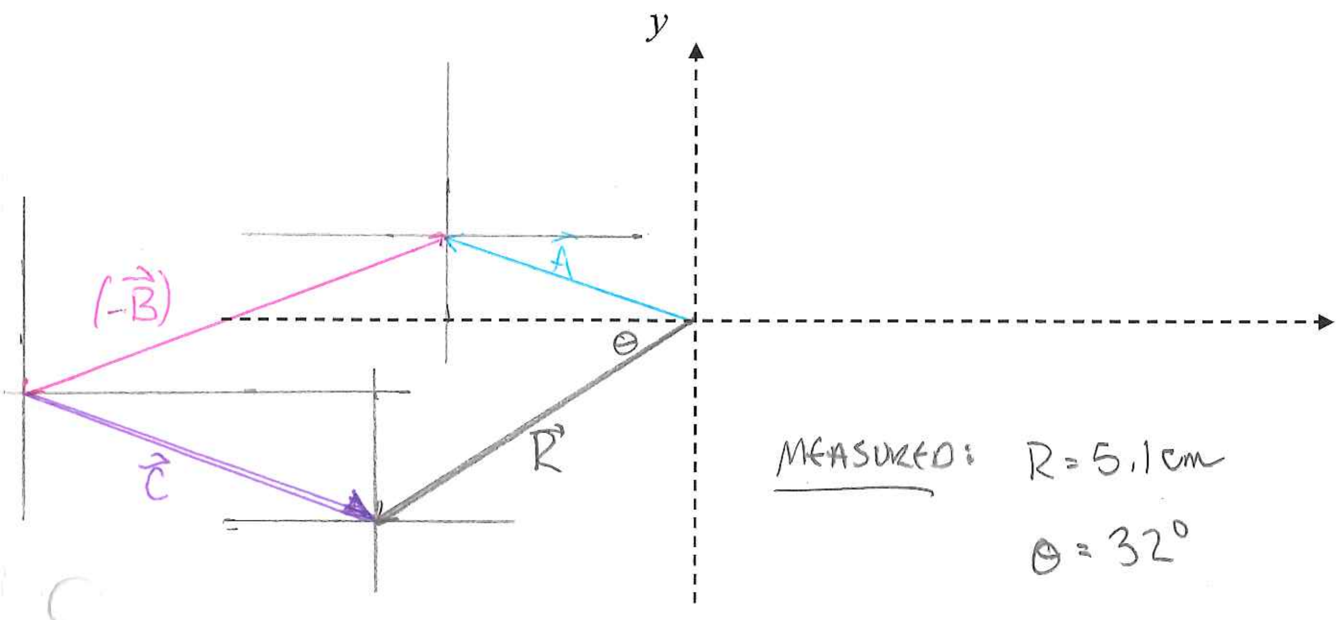
$\vec{R} = 7.15$ cm

$\theta = \tan^{-1}\left(\frac{1.42}{7.01}\right) = 11.45°$

Using a ruler and protractor show the tip-to-tail vector addition of **A** + **B** + **C** below. Draw in the vector **R** and measure the angle θ it makes with the x axis. Confirm that it matches your answer above.



Using a ruler and protractor show the tip-to-tail vector addition of **A** − **B** + **C** below. Draw in the vector **R** and measure the angle θ it makes with the x axis. Confirm that it matches your answer above.



MEASURED: R = 5.1 cm

θ = 32°

$$\vec{A} = -3.33\hat{x} + 1.08\hat{y}$$
$$+(-\vec{B}) = -5.64\hat{x} + -2.05\hat{y}$$
$$+\ \vec{C} = 4.7\hat{x} + -1.71\hat{y}$$

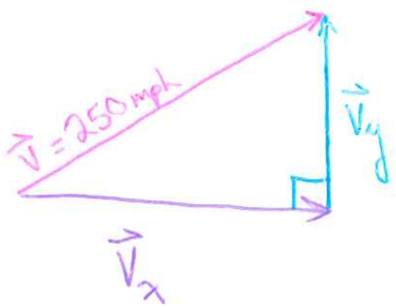
$$\vec{R} = -4.27\hat{x} - 2.68\hat{y}$$

$$R = \sqrt{4.27^2 + 2.68^2} = 5.04\,\text{cm}$$

$$\theta = \tan^{-1}\left(\frac{2.68}{4.27}\right) = 32.1°$$

Vector Problems with Physics Applications

1. A plane is flying at an angle of 35^0 to the horizontal ground at a speed of 250 mi/hr. If the sun is directly overhead shining straight down, how fast is the shadow of the plane moving along the ground. That is, calculate the x-component of the speed of the plane.



$\vec{v}_x = 250\cos 35^\circ$

$\boxed{\vec{v}_x = 204.8 \text{ mph } \hat{x}}$

2. A man walks 2 miles east, 3 miles north, and then 7 miles 30^0 north of west. What was his displacement in both **magnitude and direction** and **component** form? Calculate your answer both analytically and with a ruler and protractor.

$\vec{A} = 2\text{mi}\ \hat{x} + 0\text{mi}\ \hat{y}$

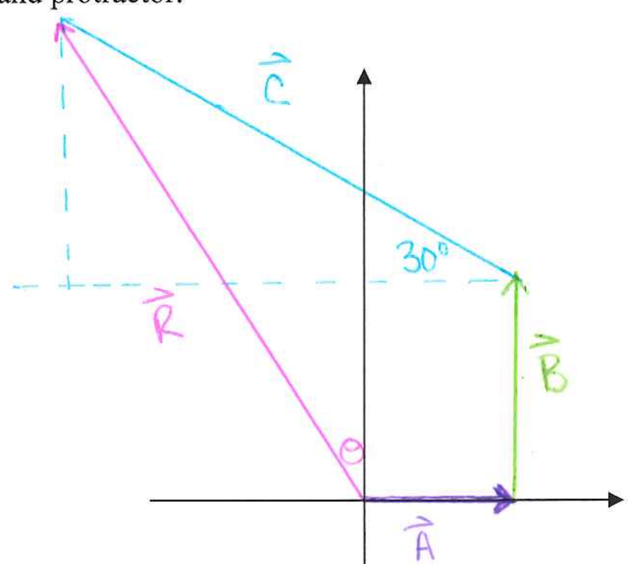
$\vec{B} = 0\text{mi}\ \hat{x} + 3\text{mi}\ \hat{y}$

$\vec{C} = -7\cos 30\ \hat{x} + 7\sin 30\ \hat{y}$

$\vec{R} = -4.06\text{mi}\ \hat{x} + 6.5\text{mi}\ \hat{y}$

$\vec{R} = \sqrt{4.06^2 + 6.5^2} = 7.7\text{ mi}$

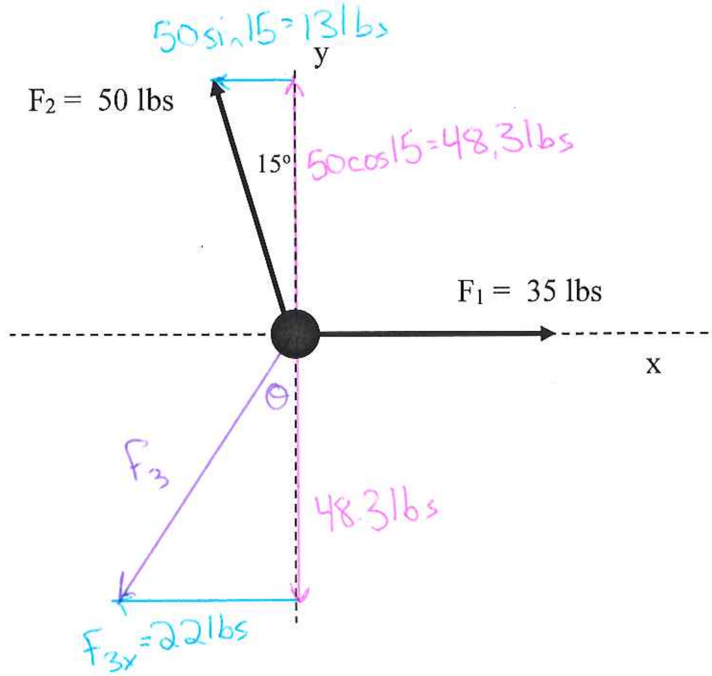
$\theta = \tan^{-1}\left(\frac{4.06}{6.5}\right) = 32^\circ$



$\vec{R} = 7.5\text{mi}$

$\theta = 33^\circ$ W of N

3. A pole has 3 ropes tied to it with the forces shown for 2 of the ropes. Calculate the force and the angle θ that would be required of the 3[rd] rope so that no resultant force is exerted on the pole.



$F_{1x} = F_{2x} + F_{3x}$

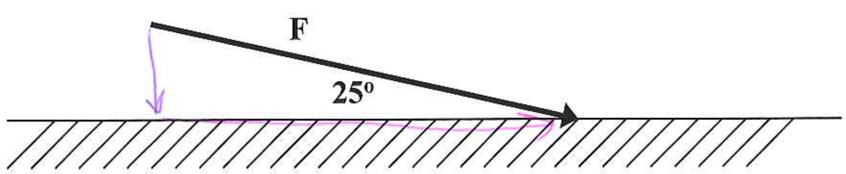
$35 = 13 + F_{3x} \rightarrow F_{3x} = 22\text{lbs}$

$F_{2y} = F_{3y} \rightarrow F_{3y} = 48.3\text{lbs}$

$F_3 = \sqrt{22^2 + 48.3^2} = \boxed{53\text{lbs}}$

$\theta = \tan^{-1}\left(\frac{22}{48.3}\right) = \boxed{24.5^\circ}$

4. If a force **F** of 300 lbs is pushing on a flat surface at an angle of 25° as shown, how much of the force is acting along the surface and how much of the force is pushing directly down into the surface.



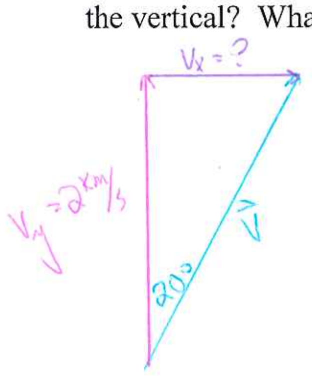
$F_y = 300\sin25^\circ$

$F_y = 126.7\text{lbs}\,\hat{y}$

$F_x = 300\cos25$

$F_x = 271.91\text{lbs}\,\hat{x}$

5. A rocket is moving vertically upward with a speed of 2 km/s. How fast to the right would the thrusters have to make the rocket go if you want the rocket to travel at 20° to the vertical? What would be the overall speed of the rocket at that point?



$\tan20 = \frac{V_x}{2} \rightarrow \boxed{V_x = .73\,\text{km/s}}$

$V = \sqrt{2^2 + .73^2} = \boxed{2.13\,\text{km/s}}$